Outline for January 23, 2001

1. Greetings and felicitations!
   a. No class on January 25 or January 30; no office hours on Wednesday, January 24 or Monday, January 29
   b. Extra office hour: Friday January 26: 11 AM–1 PM
2. Distributed system?
   a. What is it?
   b. Why use it?
3. System Architectures
   a. minicomputer mode
   b. workstation model
   c. processor pool
4. Issues
   a. global knowledge
   b. naming
   c. scalability
   d. compatibility
   e. process synchronization, communication
   f. security
   g. structure
5. Networks
   a. goals
   b. message, packet, subnet, session
   c. switching: circuit, store-and-forward, message, packet, virtual circuit, dynamic routing
   d. OSI model: PDUs, layering
      i. physical: ethernet, aloha, *etc*.
      ii. data link layer: frames, parity checks, link encryption
      iii. network layer: virtual circuit vs. datagram, routing via flooding, static routes, dynamic routes, centralized routing vs. distributed routing; congestion solutions (packet discarding, isarithmic, choke packets)
      iv. transport: services provided (UDP vs. TCP), functions to higher layers, addressing schemes (flat, DNS, etc.), gateway fragmentation and reassembly
      v. session: adds session characteristics like authentication
      vi. presentation: compression, end-to-end encryption, virtual terminal
      vii. application: user-level programs
6. Clocks
   a. happened-before relation
   b. Lamport's distributed clocks: $a \rightarrow b$ means $C(a) < C(b)$
   c. Example where $C(a) < C(b)$ does *not* mean $a \rightarrow b$
   d. Vector clocks and causal relation
   e. ordering of messages so you receive them in the order sent
      i. why
      ii. for broadcast (ISIS): Birman-Schiper-Stephenson
      iii. for point to point: Schiper-Eggli-Sandoz
7. Global state
   a. Show problem of slicing state when something is in transit
   b. Define local state; $send(m_{ij}) \in LS_i$ iff time of $send(m_{ij})$ < current time of LS_i; similar for receive
   c. transit(LS_i, LS_j); inconsistent(LS_i, LS_j); consistent state is one with inconsistent set empty for all pairs LS_i, LS_j
   d. Consistent global state: Chandry-Lamport
8. Termination detection
   a. Haung

Lamport's Clocks

Introduction

Lamport's clocks keep a virtual time among distributed systems. The goal is to provide an ordering upon events within the system.

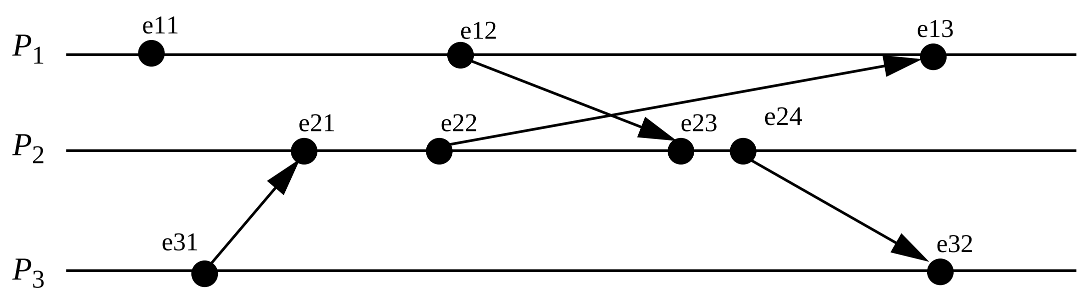
Notation

- P_i process
- C_i. clock associated with process P_i

Protocol

1. Increment clock C_i between any two successive events in process P_i: $C_i \leftarrow C_i + d$ ($d > 0$)
2. Let event a be the sending of a message by process P_i; the timestamp is $t^a = C_i(a)$ *after* the clock is incremented. Let b be the receipt of that message by P_j. Then when P_j receives the message, $C_j \leftarrow \max(C_j, t^a + d)$ ($d > 0$)

Example



Assume all clocks start at 0, and d is 1 (that is, each event incrememts the clock by 1). The events and clocks are:
e_{11}: $C_1 \leftarrow 1$

e_{31}: $C_3 \leftarrow 1$; timestamp $t^{3,1}$ of message is 1

e_{21}: $C_2 \leftarrow 2$ as $t^{3,1} = 1$, after the increment $C_2 \leftarrow 1$, and $C_2 \leftarrow \max(C_2, t^{3,1} + 1) = \max(1, 1 + 1) = \max(1, 2) = 2$

e_{22}: $C_2 \leftarrow 3$; timestamp $t^{2,2}$ of message is 3

e_{12}: $C_1 \leftarrow 2$; timestamp $t^{1,2}$ of message is 2

e_{23}: $C_2 \leftarrow 4$ as $t^{1,2} = 2$, after the increment $C_2 \leftarrow 4$, and $C_2 \leftarrow \max(C_2, t^{1,2} + 1) = \max(4, 2 + 1) = \max(4, 2) = 4$

e_{24}: $C_2 \leftarrow 5$; timestamp $t^{2,4}$ of message is 5

e_{13}: $C_1 \leftarrow 4$ as $t^{2,2} = 3$, after the increment $C_1 \leftarrow 3$, and $C_1 \leftarrow \max(C_1, t^{2,2} + 1) = \max(3, 3 + 1) = \max(3, 4) = 4$

e_{32}: $C_3 \leftarrow 6$ as $t^{2,4} = 5$, after the increment $C_3 \leftarrow 2$, and $C_3 \leftarrow \max(C_3, t^{2,4} + 1) = \max(2, 5 + 1) = \max(2, 6) = 6$

Problem

Clearly, if a $\rightarrow$ b, then $C(a) < C(b)$. But if $C(a) < C(b)$, does $a \rightarrow b$?
The answer, surprisingly, is not necessarily. In the above example, $C_3(e_{31}) = 1 < 2 = C_1(e_{12})$. But e_{31} and e_{12} are causally unrelated; that is, $e_{31} \not\rightarrow e_{12}$. However, $C_1(e_{11}) = 1 < 6 = C_3(e_{32})$, and clearly $e_{11} \rightarrow e_{32}$. Hence one cannot say one way or the other.

Vector Clocks

Introduction

This is based upon Lamport's clocks, but each process keeps track of what is believes the other processes' interrnal clocks are (hence the name, vector clocks). The goal is to provide an ordering upon events within the system.

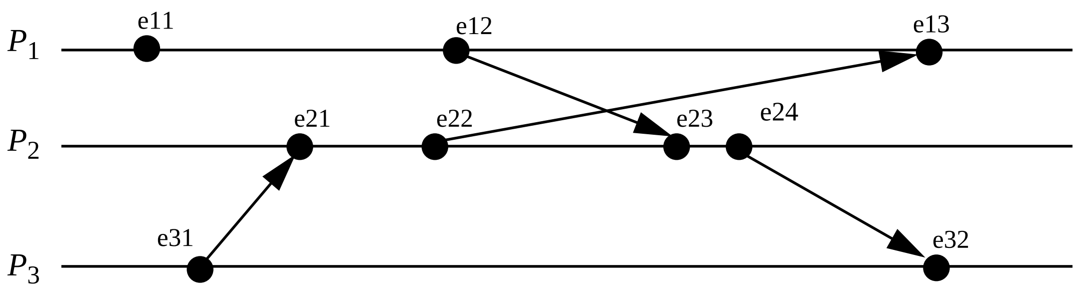
Notation

- *n* processes
- P_i process
- C_i. vector clock associated with process P_i; *j*th element is $C_i[j]$ and contains P_i's latest value for the current time in process P_k.

Protocol

1. Increment clock C_i between any two successive events in process P_i: $C_i[i] \leftarrow C_i[i] + d$ $(d > 0)$
2. Let event *a* be the sending of a message by process P_i; its vector timestamp is $t^a = C_i(a)$ *after* the clock is incremented.. Let *b* be the receipt of that message by P_j. Then when P_j receives the message, it updates its vector clock for all $k = 1, \ldots, n$: $C_j[k] \leftarrow \max(C_j[k], t^a[k])$

Example



Here is the progression of time for the three processes:
e_{11}: $C_1 \leftarrow (1, 0, 0)$

e_{31}: $C_3 \leftarrow (0, 0, 1)$; timestamp $t^{3,1}$ of message is (0, 0, 1)

e_{21}: $C_2 \leftarrow (0, 1, 1)$ as $t^{3,1} = (0, 0, 1)$, $C_2 \leftarrow (0, 1, 0)$ after the increment, $C_2[0] \leftarrow \max(C_2[0], t^{3,1}[0]) = \max(0, 0) = 0$, $C_2[1] \leftarrow \max(C_2[1], t^{3,1}[1]) = \max(1, 0) = 1$, and $C_2[2] \leftarrow \max(C_2[2], t^{3,1}[2]) = \max(0, 1) = 1$

e_{22}: $C_2 \leftarrow (0, 2, 1)$; timestamp $t^{2,1}$ of message is (0, 2, 1)

e_{12}: $C_1 \leftarrow (2, 0, 0)$; timestamp $t^{1,1}$ of message is (2, 0, 0)

e_{23}: $C_2 \leftarrow (2, 3, 1)$ as $t^{1,1} = (2, 0, 0)$, $C_2 \leftarrow (0, 3, 1)$ after the increment, $C_2[0] \leftarrow \max(C_2[0], t^{1,1}[0]) = \max(0, 2) = 2$, $C_2[1] \leftarrow \max(C_2[1], t^{1,1}[1]) = \max(3, 0) = 3$, and $C_2[2] \leftarrow \max(C_2[2], t^{1,1}[2]) = \max(0, 1) = 1$

e_{24}: $C_2 \leftarrow (2, 3, 1)$; timestamp $t^{2,2}$ of message is (2, 3, 1)

e_{13}: $C_1 \leftarrow (3, 2, 1)$ as $t^{2,1} = (0, 2, 1)$, $C_1 \leftarrow (3, 0, 0)$ after the increment, $C_1[0] \leftarrow \max(C_1[0], t^{2,1}[0]) = \max(3, 0) = 3$, $C_1[1] \leftarrow \max(C_1[1], t^{2,1}[1]) = \max(2, 0) = 2$, and $C_1[2] \leftarrow \max(C_1[2], t^{2,1}[2]) = \max(1, 0) = 1$

e_{32}: $C_3 \leftarrow (2, 3, 2)$ as $t^{2,2} = (2, 3, 1)$, $C_3 \leftarrow (0, 0, 2)$ after the increment, $C_3[0] \leftarrow \max(C_3[0], t^{2,2}[0]) = \max(2, 0) = 2$, $C_3[1] \leftarrow \max(C_3[1], t^{2,2}[1]) = \max(3, 0) = 3$, and $C_3[2] \leftarrow \max(C_3[2], t^{2,2}[2]) = \max(1, 2) = 2$

Notice that $C_1(e_{11}) < C_3(e_{32})$, so $e_{11} \rightarrow e_{32}$, but $C_1(e_{11})$ and $C_3(e_{31})$ are incomparable, so e_{11} and e_{31} are concurrent.

Birman-Schiper-Stephenson Protocol

Introduction

The goal of this protocol is to preserve ordering in the sending of messages. For example, if *send*(m_1) $\rightarrow$ send(m_2), then for all processes that receive both m_1 and m_2, *receive*(m_1)$\rightarrow$ *receive*(m_2). The basic idea is that m_2 is not given to the process until m_1 is given. This means a buffer is needed for pending deliveries. Also, each message has an associated vector that contains information for the recipient to determine if another message preceded it. Also, we shall assume all messages are broadcast. Clocks are updated only when messages are sent.

Notation

- n processes
- P_i process
- C_i. vector clock associated with process P_i; jth element is $C_i[j]$ and contains P_i's latest value for the current time in process P_k.
- t^m vector timestamp for message m (stamped *after* local clock is incremented)

Protocol

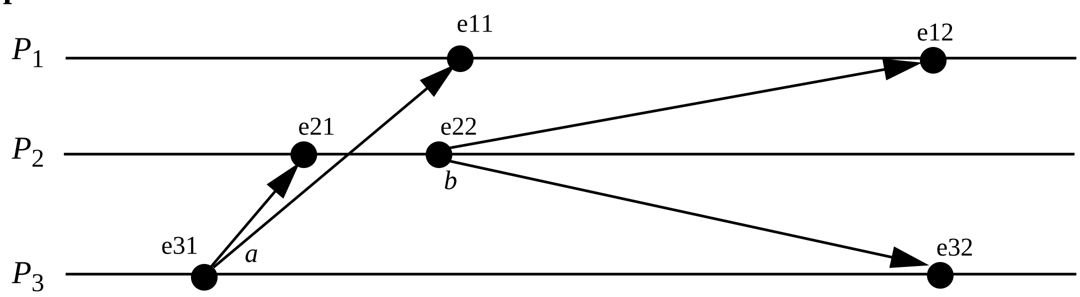
P_i broadcasts a message

1. P_i increments $C_i[i]$ and sets the timestamp $t^m = C_i$ for message m.

P_j receives a message from P_i

1. When P_j, $j \neq i$, receives m with timestamp t^m, it delays the message's delivery until *both*:
   a. $C_j[i] = t^m[i] - 1$; and
   b. for all $k \leq n$ and $k \neq i$, $C_j[k] \geq t^m[k]$.
2. When the message is delivered to P_j, update P_j's vector clock for all $k = 1, \ldots, n$: $C_j[k] \leftarrow \max(C_j[k], t^a[k])$
3. Check buffered messages to see if any can be delivered.

Example



Here is the protocol applied to the above situation:

e_{31}: P_3 sends message *a*; $C_3 = (0, 0, 1)$; $t^a = (0, 0, 1)$

e_{21}: P_2 receives message *a*. As $C_2 = (0, 0, 0)$, $C_2[3] = t^a[3] – 1 = 1 – 1 = 0$ and $C_2[1] \geq t^a[1]$ and $C_2[2] \geq t^a[2] = 0$. So the message is accepted, and C_2 is set to (0, 0, 1)

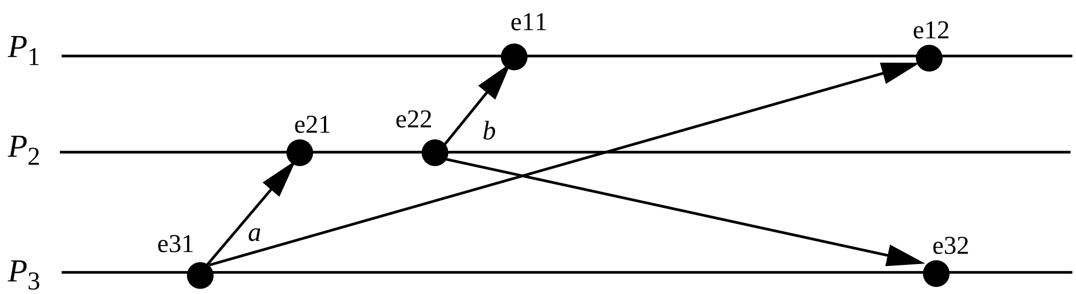
e_{22}: P_2 sends message *b*; $C_2 = (0, 1, 1)$; $t^b = (0, 1, 1)$

e_{11}: P_1 receives message *a*. As $C_1 = (0, 0, 0)$, $C_1[3] = t^a[3] – 1 = 1 – 1 = 0$ and $C_1[1] \geq t^a[1]$ and $C_1[2] \geq t^a[2] = 0$. So the message is accepted, and C_1 is set to (0, 0, 1)

e_{12}: P_1 receives message *b*. As $C_1 = (0, 0, 1)$, $C_1[2] = t^b[2] – 1 = 1– 1 = 0$ and $C_1[1] \geq t^b[1]$ and $C_1[3] \geq t^b[2] = 0$. So the message is accepted, and C_1 is set to (0, 1, 1)

e_{32}: P_3 receives message *b*. As $C_3 = (0, 0, 1)$, $C_3[2] = t^b[2] – 1 = 1 – 1 = 1$ and $C_1[1] \geq t^b[1]$ and $C_1[3] \geq t^b[2] = 0$. So the message is accepted, and C_3 is set to (0, 1, 1)

Now, suppose t^a arrived as event e_{12}, and t^b as event e_{11}:



Then the progression of time in P_1 goes like this:

e_{31}: P_3 sends message *a*; $C_3 = (0, 0, 1)$; $t^a = (0, 0, 1)$

e_{21}: P_2 receives message *a*. As $C_2 = (0, 0, 0)$, $C_2[3] = t^a[3] – 1 = 1 – 1 = 0$ and $C_2[1] \geq t^a[1]$ and $C_2[2] \geq t^a[2] = 0$. So the message is accepted, and C_2 is set to (0, 0, 1)

e_{22}: P_2 sends message *b*; $C_2 = (0, 1, 1)$; $t^b = (0, 1, 1)$

e_{11}: P_1 receives message *b*. As $C_1 = (0, 0, 0)$, $C_1[2] = t^b[2] – 1 = 1 – 1 = 0$ and $C_1[1] \geq t^b[1]$, but $C_1[3] < t^b[3]$, so the message is held until another message arrives. The vector clock updating algorithm is not run.

e_{12}: P_1 receives message *a*. As $C_1 = (0, 0, 0)$, $C_1[3] = t^a[3] – 1 = 1 – 1 = 0$, $C_1[1] \geq t^a[1]$, and $C_1[2] \geq t^a[2]$. The message is accepted and C_1 is set to (0, 0, 1). Now the queue is checked. As $C_1[2] = t^b[2] – 1 = 1 – 1 = 0$, $C_1[1] \geq t^b[1]$, and $C_1[3] \geq t^b[3]$, that message is accepted and C_1 is set to (0, 1, 1).

e_{32}: P_3 receives message *b*. As $C_3 = (0, 0, 1)$, $C_3[2] = t^b[2] – 1 = 1 – 1 = 1$ and $C_1[1] \geq t^b[1]$ and $C_1[3] \geq t^b[2] = 0$. So the message is accepted, and C_3 is set to (0, 1, 1)

Schiper-Eggli-Sandoz Protocol

Introduction

The goal of this protocol is to ensure that messages are given to the receiving processes in order of sending. Unlike the Birman-Schiper-Stephenson protocol, it does not require using broadcast messages. Each message has an associated vector that contains information for the recipient to determine if another message preceded it. Clocks are updated only when messages are sent.

Notation

- n processes
- P_i process
- C_i. vector clock associated with process P_i; jth element is $C_i[j]$ and contains P_i's latest value for the current time in process P_k.
- t^m vector timestamp for message m (stamped *after* local clock is incremented)
- t^i current time at process P_i
- V_i vector of P_i's previously sent messages; $V_i[j] = t^m$, where the last message sent to P_j has the vector timestamp t^m; $V_i[j][k]$ is the kth component of $V_i[j]$.
- V^m vector accompanying message m

Protocol

P_i sends a message to P_j

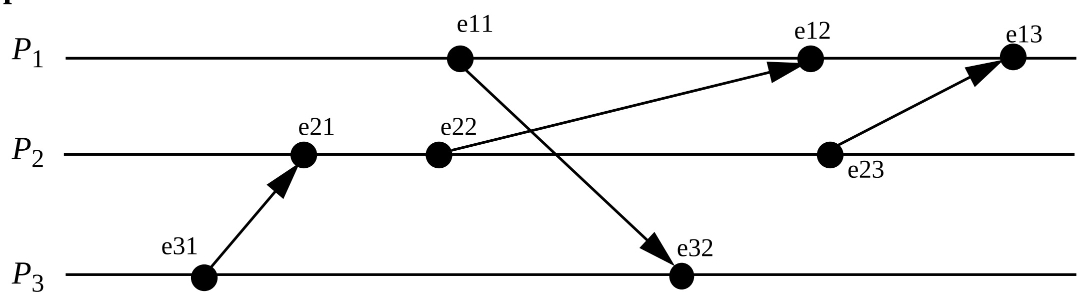
1. P_i sends message m, timestamped t^m, and V_i, to process P_j .
2. P_i sets $V_i[j] \leftarrow t^m$.

P_j receives a message from P_i

1. When P_j, $j \neq i$, receives m, it delays the message's delivery if *both*:
   a. $V^m[j]$ is set; and
   b. $V^m[j] < t^j$

   Otherwise it is queued for later delivery.
2. If the message can be delivered to P_j, the following three actions occur:
   a. Update all set elements of V_j with the corresponding elements of V^m, except for $V_j[j]$, as follows:
      i. If $V_j[k]$ and $V^m[k]$ are uninitialized, do nothing.
      ii. If $V_j[k]$ is uninitialized and $V^m[k]$ is initialized, set $V_j[k] \leftarrow V^m[k]$.
      iii. If both $V_j[k]$ and $V^m[k]$ are initialized, $V_j[k][k'] \leftarrow \max(V_j[k][k'], V^m[k][k'])$ for all $k' = 1, \ldots, n$
3. Update P_j's vector clock for all $k = 1, \ldots, n$: $C_j[k] \leftarrow \max(C_j[k], t^m[k])$
4. Check buffered messages to see if any can be delivered.

Example



Here is the protocol applied to the above situation. [...] and { ... } are like (...) but used when too many parentheses would be confusing (to me, at any rate!):

e_{31}: P_3 sends message $m_{3,1}$ to P_2. C_3 = (0, 0, 1); $t^{3,1} \leftarrow$ (0, 0, 1), $V^{3,1} \leftarrow$ (?, ?, ?); $V_3 \leftarrow$ [?, (0, 0, 1), ?]

e_{21}: P_2 receives message $m_{3,1}$ from P_3. As $V^{3,1}$[2] = (?, ?, ?)[2] is uninitialized, the message is accepted.
$V_2 \leftarrow$ [?, ?, ?] and $C_2 \leftarrow$ *max*[(0, 0, 0), (0, 0, 1)] = (0, 0, 1)

e_{22}: P_2 sends message $m_{2,1}$ to P_1. $C_2 \leftarrow$ (0, 1, 1); $t^{2,1} \leftarrow$ (0, 1, 1), $V^{2,1} \leftarrow$ [?, ?, ?]; $V_2 \leftarrow$ [(0, 1, 1), ?, ?]

e_{11}: P_1 sends message $m_{1,1}$ to P_3. $C_1 \leftarrow$ (1, 0, 0); $t^{1,1} \leftarrow$ (1, 0, 0), $V^{1,1} \leftarrow$ (?, ?, ?); $V_1 \leftarrow$ [?, ?, (1, 0, 0)]

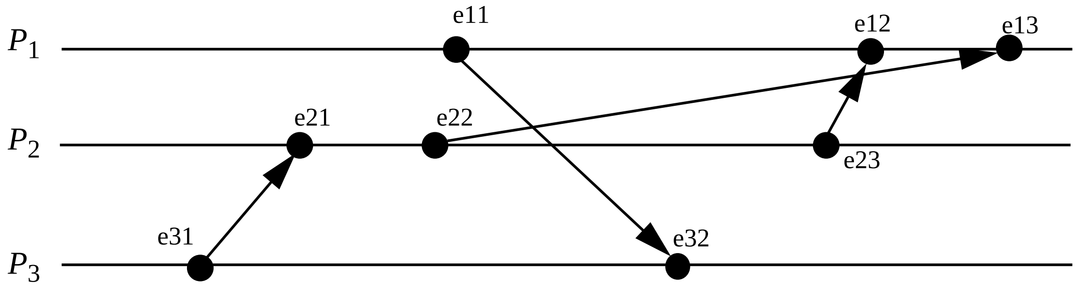
e_{32}: P_3 receives message $m_{1,1}$ from P_1. As $V^{1,1}$[3] = (?, ?, ?)[3] is uninitialized, the message is accepted.
$V_3 \leftarrow$ [?, (0, 0, 1), ?] and $C_3 \leftarrow$ *max*[(0, 0, 1), (1, 0, 0)] = (1, 0, 1).

e_{12}: P_1 receives message $m_{2,1}$ from P_2. As $V^{2,1}$[1] = [?, ?, ?][1] is uninitialized, the message is accepted.
$V_1 \leftarrow$ [?, ?, (1, 0, 0)] and $C_1 \leftarrow$ *max*[(1, 0, 0), (0, 1, 1)] = (1, 1, 1)

e_{23}: P_2 sends message $m_{2,2}$ to P_1. $C_2 \leftarrow$ (0, 2, 1); $t^{2,2} \leftarrow$ (0, 2, 1), $V^{2,2} \leftarrow$ [(0, 1, 1), ?, ?]; $V_2 \leftarrow$ [(0, 2, 1), ?, ?]

e_{13}: P_1 receives message $m_{2,2}$ from P_2. *As* $V^{2,2}$[1] = (0, 1, 1) < (1, 1, 1) = C_1, the message is accepted.
$V_1 \leftarrow$ [?, ?, (1, 0, 0)] and $C_1 \leftarrow$ *max*[(0, 2, 1), (1, 1, 1)] = (1, 2, 1)

Now, suppose $m_{2,1}$ arrived as event e_{13}, and $m_{2,2}$ as event e_{12}:



e_{31}: P_3 sends message $m_{3,1}$ to P_2. C_3 = (0, 0, 1); $t^{3,1} \leftarrow$ (0, 0, 1), $V^{3,1} \leftarrow$ (?, ?, ?); $V_3 \leftarrow$ [?, (0, 0, 1), ?]

e_{21}: P_2 receives message $m_{3,1}$ from P_3. As $V^{3,1}$[2] = (?, ?, ?)[2] is uninitialized, the message is accepted.
$V_2 \leftarrow$ [?, ?, ?] and $C_2 \leftarrow$ *max*[(0, 0, 0), (0, 0, 1)] = (0, 0, 1)

e_{22}: P_2 sends message $m_{2,1}$ to P_1. $C_2 \leftarrow$ (0, 1, 1); $t^{2,1} \leftarrow$ (0, 1, 1), $V^{2,1} \leftarrow$ [?, ?, ?]; $V_2 \leftarrow$ [(0, 1, 1), ?, ?]

e_{11}: P_1 sends message $m_{1,1}$ to P_3. $C_1 \leftarrow$ (1, 0, 0); $t^{1,1} \leftarrow$ (1, 0, 0), $V^{1,1} \leftarrow$ (?, ?, ?); $V_1 \leftarrow$ [?, ?, (1, 0, 0)]

e_{32}: P_3 receives message $m_{1,1}$ from P_1. As $V^{1,1}$[3] = (?, ?, ?)[3] is uninitialized, the message is accepted.
$V_3 \leftarrow$ [?, (0, 0, 1), ?] and $C_3 \leftarrow$ *max*[(0, 0, 1), (1, 0, 0)] = (1, 0, 1).

e_{23}: P_2 sends message $m_{2,2}$ to P_1. $C_2 \leftarrow$ (0, 2, 1); $t^{2,2} \leftarrow$ (0, 2, 1), $V^{2,2} \leftarrow$ [(0, 1, 1), ?, ?]; $V_2 \leftarrow$ [(0, 2, 1), ?, ?]

e_{12}: P_1 receives message $m_{2,2}$ from P_2. But $V^{2,2}$[1] = (0, 1, 1) $\not<$ (1, 0, 0) = C_1, so the message is queued.

e_{13}: P_1 receives message $m_{2,1}$ from P_2. As $V^{2,1}$[1] = [?, ?, ?][1] is uninitialized, the message is accepted.
$V_1 \leftarrow$ [?, ?, (1, 0, 0)] and $C_1 \leftarrow$ *max*[(1, 0, 0), (0, 1, 1)] = (1, 1, 1).

The message on the queue is now checked. As $V^{2,2}$[1] = (0, 1, 1) < (1, 1, 1) = C_1, the message is now accepted.
$V_1 \leftarrow$ [?, ?, (1, 0, 0)] and C_1 is set to (1, 2, 1).

Chandy-Lamport Global State Recording Protocol

Introduction

The goal of this distributed algorithm is to capture a consistent global state. It assumes all communication channels are FIFO. It uses a distinguished message called a *marker* to start the algorithm.

Protocol

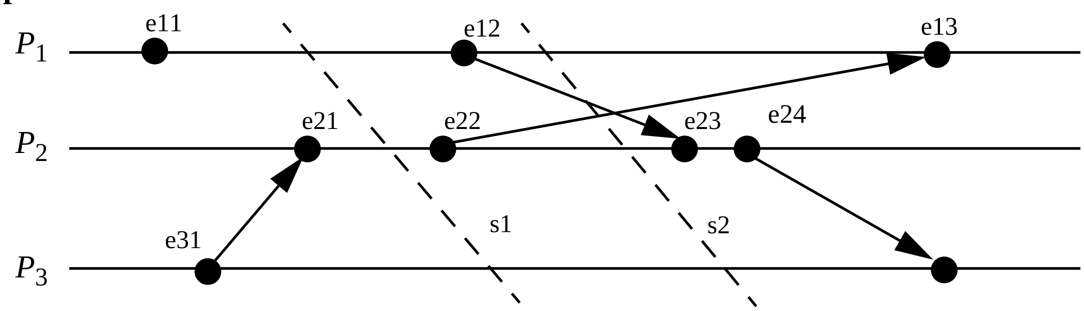
P_i sends marker

1. *Pi* records its local state LS_i
2. For each C_{ij} on which P_i has not already sent a marker, P_i sends a marker *before* sending other messages.

P_i receives marker from P_j

1. If P_i has *not* recorded its state:
   a. Record the state of C_{ji} as empty
   b. Send the marker as described above
2. If P_i has recorded its state LS_i
   a. Record the state of C_{ji} to be the sequence of messages received between the computation of LS_i and the marker from C_{ji}.

Example



Here, all processes are connected by communications channels C_{ij}. Messages being sent over the channels are represented by arrows between the processes.

Snapshot s_1:

P_1 records LS_1, sends markers on C_{12} and C_{13}

P_2 receives marker from P_1 on C_{12}; it records its state LS_2, records state of C_{12} as empty, and sends marker on C_{21} and C_{23}

P_3 receives marker from P_1 on C_{13}; it records its state LS_3, records state of C_{13} as empty, and sends markers on C_{31} and C_{32}.

P_1 receives marker from P_2 on C_{21}; as LS_1 is recorded, it records the state of C_{21} as empty.

P_1 receives marker from P_3 on C_{31}; as LS_1 is recorded, it records the state of C_{31} as empty.

P_2 receives marker from P_3 on C_{32}; as LS_2 is recorded, it records the state of C_{32} as empty.

P_3 receives marker from P_2 on C_{23}; as LS_3 is recorded, it records the state of C_{23} as empty.

Snapshot s_2: now messages are in transit on C_{12} and C_{21}.

P_1 records LS_1, sends markers on C_{12} and C_{13}

P_2 receives marker from P_1 on C_{12} after the message from P_1 arrives; it records its state LS_2, records state of C_{12} as empty, and sends marker on C_{21} and C_{23}

P_3 receives marker from P_1 on C_{13}; it records its state LS_3, records state of C_{13} as empty, and sends markers on C_{31} and C_{32}.

P_1 receives marker from P_2 on C_{21}; as LS_1 is recorded, and a message has arrived since LS_1 was recorded, it records the state of C_{21} as containing that message.

P_1 receives marker from P_3 on C_{31}; as LS_1 is recorded, it records the state of C_{31} as empty.

P_2 receives marker from P_3 on C_{32}; as LS_2 is recorded, it records the state of C_{32} as empty.

P_3 receives marker from P_2 on C_{23}; as LS_3 is recorded, it records the state of C_{23} as empty.

Huang's Termination Detection Protocol

Introduction

The goal of this protocol is to detect when a distributed computation terminates.

Notation

- n processes
- P_i process; without loss of generality, let P_0 be the *controlling agent*
- W_i. weight of process P_i; initially, $W_0 = 1$ and for all other i, $W_i = 0$.
- $B(W)$ computation message with assigned weight W
- $C(W)$ control message sent from process to controlling agent with assigned weight W

Protocol

P_i sends a computation message to P_j

1. Set W_i' and W_j to values such that $W_i' + W_j = W_i$, $W_i > 0$, $W_j > 0$. (W_i' is the new weight of P_i.)
2. Send $B(W_j)$ to P_j

P_j receives a computation message $B(W)$ from P_i

1. $W_j = W_j + W$
2. If P_j is idle, P_j becomes active

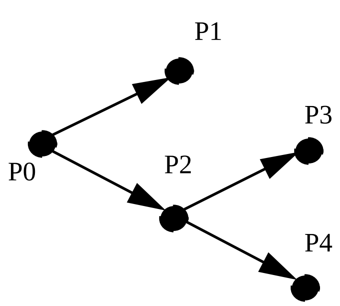
P_i becomes idle:

1. Send $C(W_i)$ to P_0
2. $W_i = 0$
3. P_i becomes idle

P_i receives a control message $C(W)$:

1. $W_i = W_i + W$
2. If $W_i = 1$, the computation has completed.

Example



The picture shows a process P_0, designated the *controlling agent*, with $W_0 = 1$. It asks P_1 and P_2 to do some computation. It sets W_1 to 0.2, W_2 to 0.3, and W_3 to 0.5. P_2 in turn asks P_3 and P_4 to do some computations. It sets W_3 to 0.1 and W_4 to 0.1.
When P_3 terminates, it sends C(W_3) = C(0.1) to P_2, which changes W_2 to 0.1 + 0.1 = 0.2.
When P_2 terminates, it sends C(W_2) = C(0.2) to P_0, which changes W_0 to 0.5 + 0.2 = 0.7.
When P_4 terminates, it sends C(W_4) = C(0.1) to P_0, which changes W_0 to 0.7 + 0.1 = 0.8.
When P_1 terminates, it sends C(W_1) = C(0.2) to P_0, which changes W_0 to 0.8 + 0.2 = 1.
P_0 thereupon concludes that the computation is finished.
Total number of messages passed: 8 (one to start each computation, one to return the weight).